• Gibbard Satterthwaite impossibility
  – For $|O| \geq 3$ and when “all strict preference orders” are possible, then SP and onto $\Rightarrow$ dictatorial.

• Work arounds:
  – **Restrict preference domain**
  – Two outcomes
  – Drop strategy-proofness
Restricted Preference Domains

• Two-sided matching
  – Outcome?
  – All strict preferences possible?

• House allocation (initial property rights)
  – What is an outcome?
  – All strict preference orders possible?

• Paired-Kidney Donation
  – What is an outcome?
  – All strict preference orders possible?

Review

• Deferred acceptance (2-sided matching)
  – Truthful reporting is dominant strategy for “proposing” side
  – Stable (for reported preference orders)
  – Stable matchings have lattice structure

• Randomized Serial Dictatorship (1-sided)
  – Strategy-proof, anonymous, and Pareto optimal.
II. One-sided Matching

• “House allocation” problem
  – No initial allocation

• “Housing market” problem
  – Initial allocation of items

Housing Market Problem

Each agent has a house.

Strict preference order on houses.

(Hartline/Immorlica)
Housing Markets

Question: Can we find trades to make all agents better off?

Top-Trading Cycle (TTC) mechanism

1. Each agent “points” to most preferred house (allow self-edge).
2. Trade on cycles, agents and houses leave market.
3. Each remaining agent points to most preferred, remaining house.
4. Repeat (#2, #3), until no agents left.
TTC mechanism

(Hartline/Immorlica)

TTC mechanism

(Hartline/Immorlica)
TTC mechanism

What if there are no cycles? What if an agent is on multiple cycles?
Top trading cycles

1. Each agent points to her most preferred house that is still on the market.
2. Select a cycle and trade, agents that trade leave the market.

(Hartline/Immorlica)
Strategy-proof
Pareto optimal
Core: no subset prefer to trade amongst themselves

Uses of TTC

- Trading books, art work, furniture
- Vacation house exchange

**

- A variation on TTC (also needs to handle school priorities) is used for school choice in San Francisco, and has been used in New Orleans
- Can provide improved welfare properties, but not stable.

**

- Core is empty even if only two types of items, with unit-demand preferences.
III: Kidney-Paired Donation

Kidney disease

- Kidney failure serious medical problem
- Preferred treatment: kidney transplant
  - Cadaver kidneys
  - Donation from live patient
  - Must be blood-type and tissue-type compatible
At this time:

114,675 people waiting for an organ transplant in the US.

http://optn.transplant.hrsa.gov

In 2008,

10,526 patients received cadaver kidneys.

4,857 patients received live donor kidneys.

5,920 patients died or became too sick for a transplant.

(Hartline/immortica)
Econ 101

- Trade for $$s

I have an extra kidney.

I need a kidney. My value for it is my value for my life.

(Hartline/Immorlica)

Repugnance

(Roth)

Often $x + $ is repugnant, even when $x$ alone is not.
"We didn’t have time to pick up a bottle of wine, but this is what we would have spent."

(Hartline/Immortica)

Illegal, also.

Section 301 of the National Organ Transplant Act, “Prohibition of organ purchases” imposes criminal penalties on any person who

“knowingly acquire[s], receive[s], or otherwise transfer[s] any human organ for valuable consideration for use in human transplantation”

(Hartline/Immortica)
Kidney-Paired Donation

Incompatible **pairs** arrive into market.
- a donor and a patient
- participate in swaps, cycles…

Trade “like for like.” Legal.

Compatibility

- Tissue type (crossmatch test)
Swap (2-cycle)

Kidney exchanges (APD, UNOS, NKR)

1. Pairs register in database.
2. Exchange forms a graph, representing possible compatibilities
3. Feasible matchings found
4. Additional medical tests ("cross match")
5. Transplants performed
Match Offer Process

Modeling KX as a Housing market?

• Each pair “points” to its most preferred other pair.
• Find cycles. Repeat.
Different, because:

- 0/1 preferences (... roughly speaking, a kidney is compatible or not)
- Limits on cycle sizes:
  - Ethics. What if someone changes their mind?
    → do all transplants on cycle at the same time
  - Last minute tests may reveal incompatibility
- May be a weighted objective (medical priorities)

Kidney-Paired Donation

- Find vertex-disjoint cycles of length \( \leq k \) that cover as many vertices as possible

(k=2)  
(k=3)  
(k=3)

(Conitzer)
Special case: k=2

• Step 1: replace bidirectional edges with an undirected edge.
• Step 2: drop other edges.

1 \leftrightarrow 2 \leftrightarrow 3 \leftrightarrow 4

• Step 3: find \textit{max cardinality matching} (max #edges, every vertex incident on at most one edge)
• Edmond’s algorithm (polynomial time.)

Complexity (max vertex-disjoint cycles)

• k = 2: in \textbf{P} by Edmonds alg
• k = \infty in \textbf{P}
K=\infty: Edge formulation

\begin{align*}
\max_y & \sum_{(i,j) \in E} y_{ij} \\
\text{s.t.} & \sum_j y_{ij} \leq 1, \quad \forall i \\
& \sum_j y_{ij} = \sum_i y_{ji}, \quad \forall i \\
& y_{ij} \geq 0
\end{align*}

Optimal solutions non-fractional!

Complexity

- \( k = 2 \): in \textbf{P} by Edmonds
- \( k = \infty \) in \textbf{P}
- But \( k = 3, 4, 5, \ldots \): \textbf{NP}-hard!
Integer Programming Formulation

• $y_c \in \{0,1\}$ for each cycle
  
  max $\sum_c |c| y_c$

  s.t. $\sum_{c: i \in c} y_c \leq 1$ for all pairs $i$

• For $k=3$, $n=5000$ pairs, $4 \times 10^8$ cycles (!)

• Can bring in new variables $y_c$ on the fly
  (“column generation”, “branch and price”)
Would like to allow for longer (non-simultaneous) cycles

• How? Ethical concerns?

Altruistic Non-simultaneous Donor Chains
Altruistic Non-simultaneous Donor Chains

• “Good Samaritan” donors. Enable chains

National Kidney Register

Completed 2900+ transplants
Entirely “altruistic non-simultaneous chains”

• 60 person, 30 transplant chain (8/11- 12/11)

Matching with altruistic donors

How to solve the “cycles and chains” problem?

Add zero weight edges pointing back to altruistic donors
Matching with altruistic donors

- Reduce to cycles problem. Still need length constraint; and, long cycles must include D*.
- As of 2016: can solve ~100 pairs with k<= 20
- Computationally challenging!

Practical considerations

- Hospital incentives (pooling donor lists)
- Medical objective (life-years, immunological status, etc.)
- Patient-donor pairs joining multiple exchange programs
- Unknown tissue-type incompatibility
- Dynamic matching (tradeoffs)
- Computational difficulty