Prediction market

- A financial market designed for information aggregation and prediction
- Trade contracts, with payoff associated with an observed outcome in future
Basic Idea

• Construct a **contract** on outcome $o \in \{0, 1\}$
• Pay $1$ if Warren elected ($o=1$), $0$ if Trump elected ($o=0$)

• Trader:
  – If belief $p = \Pr(o=1)$, then value $p$ for contract
  – Buy if $p > \text{price}$, sell if $p < \text{price}$
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- Idea: equilibrium price represents posterior belief, given dispersed information
## Financial vs Prediction Markets

<table>
<thead>
<tr>
<th></th>
<th>Financial</th>
<th>Prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Primary use</strong></td>
<td>Capital allocation</td>
<td>Information aggregation</td>
</tr>
<tr>
<td></td>
<td>Hedge risk</td>
<td></td>
</tr>
<tr>
<td><strong>Secondary use</strong></td>
<td>Information aggregation</td>
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</table>

1 person

- (verification)
  - elicit belief
  - scoring rule

n participants

- (no verification)
  - elicit signal
  - prediction market
  - peer prediction
Non-market alternatives

- Opinion poll
  - Sampling
  - No incentive to be truthful
  - Equally weighted information
  - Hard to be real-time
Non-market alternatives

- Opinion poll
  - Sampling
  - No incentive to be truthful
  - Equally weighted information
  - Hard to be real-time
- Ask Experts
  - Identifying experts can be hard
  - Incentives
  - Combining opinions can be difficult

- Prediction Markets
  - Self-selection
  - Monetary incentive
  - Money-weighted information
  - Real-time
  - Self-organizing
Applications

- Betfair, PredictIt, Hypermind. Aggregated by Predictwise
- HP, Google: private markets for sales forecasts
- Iowa Electronic Market
- CMU Gates-Hillman prediction market
- HSX prediction market movie box office
- CultivateLabs, ConsensusPoint
- https://www.crowdmed.com/
Empirical studies.

- Racetrack betting odds beat track experts (Figlewski 79)
- Orange juice futures improve weather forecast (Roll 84)
- IEM beat opinion polls 451/596 times (Berg 01, Pennock 02)
- HP prediction market beat internal sales forecasts 6/8 times (Plott 00)

Iowa Electronic Market 1992 Election

[Source: Berg, DARPA Workshop, 2002]
IEM versus Polls: 1996
(Berg, Nelson and Rietz, 2001)

Predictive Accuracy
Berg, Forsythe, Nelson and Rietz (2001)

[Source: Berg, DARPA Workshop, 2002]
IEM Information Revelation Through Time

(Past 4 presidential elections)


[Source: Wolfers & Zitzewitz 2004]

Predicting Movie Success

[Source: Wolfers & Zitzewitz 2004]
Different Kinds of Contracts

• Binary (“Warren” or “Trump”).
  – winner-take all contract ($1 if EW)
• Continuous (vote share)
  – Index contract (pay $1 * vote share)
• Multi-valued (“Warren” or “Biden” or “Sanders”), then need multiple contracts.
  – $1 if EW. $1 if JB. $1 if BS.

Market Designs

• Call market
• Continuous Double Auction (CDA)
  – Betfair
  – HSX
• Automated Market Maker (LMSR)
  – CultivateLabs (was Inkling markets)
  – CrowdMed
Desirable Properties

- Liquidity (can always trade any quantity, without moving the price too much)
- Information aggregation
- No arbitrage

Design 1: A Call market

- **Buy offers (N=4)**
  - $0.15
  - $0.12
  - $0.09
  - $0.05

- **Sell offers (M=5)**
  - $0.30
  - $0.17
  - $0.13
  - $0.11
  - $0.08
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<tr>
<th>Price</th>
<th>≤0.05</th>
<th>(0.05, 0.08)</th>
<th>(0.08, 0.09)</th>
<th>(0.09, 0.11)</th>
<th>(0.11, 0.12)</th>
<th>(0.12, 0.13)</th>
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<th>≥0.30</th>
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<tr>
<td>Demand</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
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</tr>
<tr>
<td>Supply</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
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Short Selling in a CDA

- Contract “$1 if 201 warmest on record”
- Bids $0.5, $0.6, $0.7. Asks?
Short Selling in a CDA

• Contract “$1 if 2019 warmest on record”
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• Short sell: Borrow a contract to sell

• Ask $0.65. Trade with Bid $0.7 at $0.7.
Short Selling in a CDA

- Contract “$1 if 2019 warmest on record”
- Bids $0.5, $0.6, $0.7. Asks?
- *Short sell: Borrow a contract to sell*
- Ask $0.65. Trade with Bid $0.7 at $0.7
  - Short seller must later buy a contract (e.g., at price $0.3, or price $1!)

- #contracts owned = #contracts short
CDAs can have Low Liquidity

Design 3: Automated Market Maker

- Market maker quotes a price to buy or sell any quantity
- Goal: improve liquidity, and thus information aggregation
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<td>Buy 2 @ lose</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>Sell 1 @ win</td>
<td>6</td>
<td>2</td>
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- **Risk**: may run at a loss
Desirable Properties

• No round-trip arbitrage
• Prices strictly positive, sum to one
• Responsiveness (buy->price up, sell->)
• Liquidity (trade any quantity, don’t move price very much).
• Myopic incentives
• Bounded loss to market-maker

Cost-based market maker

• Market state \( x \in \mathbb{R}^m_{\geq 0} \)
  \( x_k \) sold of contract \( k \)
  may be fractional

• Cost function \( C(x) \)
  \( C(x) - C(0) \) is total payment to AMM

• Suppose outcomes are (win, lose)
  Buy 10 @ win. Trader pays \( C(10,0) - C(0,0) \)
  Buy 5 @ lose. Trader pays \( C(10,5) - C(10,0) \)
  Sell 2 @ win. Trader gets \( C(10,5) - C(8,5) \)
  Buy 3 @ win. Trader pays \( C(11,5) - C(8,5) \).

• No round-trip arbitrage.
• Worst-case loss: \( \text{Loss}(x) = \left( \max_j x_j \right) - \left( C(x) - C(0) \right) \)
Example: LMSR Cost Function

\[ C(x_0, x_1) = \beta \ln(e^{x_0/\beta} + e^{x_1/\beta}) \]

- Change in market state when buy \( Q_k \)

\[ x \mapsto (x_k + Q_k, x_{-k}) \]
• Change in market state when buy $Q_k$

\[ x \mapsto (x_k + Q_k, x_{-k}) \]

• Total payment by trader:

\[ \pi_k^{\text{buy}} (x; Q_k) = C(x_k + Q_k, x_{-k}) - C(x) \]

• Price for an infinitesimal amount:

\[ \pi_k (x) = \frac{\partial}{\partial x_k} C(x) \]
• Change in market state when buy $Q_k$

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• Equivalent expression for

$$\pi_k^{\text{buy}}(x; Q_k) = \int_{z=x_k}^{x_k + Q_k} \pi_k(z, x_{-k}) \mathrm{d}z$$

### Price Quotes

Cost function

$$C(x_0, x_1) = \beta \ln(e^{x_0/\beta} + e^{x_1/\beta})$$

Instantaneous price

$$\pi_k(x) = \frac{\partial}{\partial x_k} C(x)$$

$$\pi_0(x) = \frac{e^{x_0/\beta}}{e^{x_0/\beta} + e^{x_1/\beta}}$$

$$\pi_1(x) = \frac{e^{x_1/\beta}}{e^{x_0/\beta} + e^{x_1/\beta}}$$

Strictly positive. Sum to one. Responsive. High liquidity as $\beta \to \infty$.

Left to show…. myopic incentives and Bounded loss.
Myopic incentives

<table>
<thead>
<tr>
<th>State</th>
<th>Price(win)</th>
<th>Belief</th>
<th>Trade</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x0,x1)</td>
<td>$0.6</td>
<td>0.7</td>
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Later:
- Harvard win: Profit = $Q_0 - \text{cost trade}
- Harvard lose: Loss = \text{cost trade}

• Claim: profit/loss exactly same as if reporting 0.7 to a scoring rule!
• => myopic incentives

Market Scoring Rules

• Sequence of reports \( q^{(0)}, q^{(1)}, \ldots, q^{(n)} \)
• \( q^{(0)} \) is uniform
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- $q^{(0)}$ is uniform
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- Log scoring rule. Strictly proper? Yes!

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• Upon realization of \( o_k \), pay agent \( i \):
  \[ t(q^{(i)}, o_k) - t(q^{(i-1)}, o_k) \]
• Log scoring rule. Strictly proper? Yes!
• Total cost = \( t(q^{(n)}, o_k) - t(q^{(0)}, o_k) \)
  \[ \leq \beta \ln(1) - \beta \ln(1/m) = \beta \ln(m) \]

Claim: profit/loss exactly same as LMSR
– myopic incentives, bounded loss follows.
Equivalence

- Automated market maker. Trader buys $Q_0$. Profit to trader if outcome 0 occurs:

$$Q_0 - (C(x_0 + Q_0, x_1) - C(x)) = Q_0 - \beta \left[ \ln \left( e^{(x_0+Q_0)/\beta} + e^{x_1/\beta} \right) - \ln \left( e^{x_0/\beta} + e^{x_1/\beta} \right) \right]$$

- Now a market scoring rule. Report belief $Q_0$. Profit to trader if outcome 0 occurs:

$$\beta \ln \left( \frac{e^{(x_0+Q_0)/\beta}}{e^{(x_0+Q_0)/\beta} + e^{x_1/\beta}} \right) - \beta \ln \left( \frac{e^{x_0/\beta}}{e^{x_0/\beta} + e^{x_1/\beta}} \right)$$
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$$\beta \ln \left( \frac{e^{(x_0+Q_0)/\beta}}{e^{(x_0+Q_0)/\beta} + e^{x_1/\beta}} \right) - \beta \ln \left( \frac{e^{x_0/\beta}}{e^{x_0/\beta} + e^{x_1/\beta}} \right) = x_0 + Q_0 - x_0 - \beta \left[ \ln \left( \frac{e^{(x_0+Q_0)/\beta}}{e^{x_0/\beta} + e^{x_1/\beta}} \right) - \ln \left( \frac{e^{x_0/\beta}}{e^{x_0/\beta} + e^{x_1/\beta}} \right) \right].$$
• Conclude that a cost-based market maker with rule

\[ C(x) = \beta \ln \left( \sum_{j=0}^{m-1} e^{x_j / \beta} \right) \]

satisfies all desirable properties!

• Used, e.g. by CultivateLabs (was Inklingmarkets)

Following is Advanced material

• (Optional!)
Combinatorial Markets

• Suppose want to predict:
  – The outcome of every game in the NCAA basketball tournament: $2^{63}$
  – The outcome of every state (and DC) in the US presidential election: $2^{51}$

Example: Y! Predictalot

D. Pennock
Example: Y! Predictalot

9.2 quintillion outcomes

Combinatorial Pred Markets

• Can express dependencies
  • “If Harvard wins first two games, then probability win 3rd game = 0.6”
  • “If Republicans win OH, then probability win PA = 0.7”

• Can still predict marginal probabilities; e.g.
  – probability Harvard gets to the semi-final
  – probability that Warren wins the nomination
Combinatorial Pred Markets

• Can’t use a CDA. Why?

• Could try to use an AMM.
  – Need to be able to represent state on an exponential number of outcomes
Combinatorial Pred Markets

• Can’t use a CDA. Why?

• Could try to use an AMM.
  – Need to be able to represent state on an exponential number of outcomes

• Also need an expressive language
  – E.g., “Dems win PA and OH, Reps win FL”
  – E.g., “Harvard gets to semi-final”
  – E.g., “Democratic party wins national election”

Trades in Comb Pred Market

• In principle, can buy contracts on any subset of outcomes (“events”)

• $|O| = 2^{63}$. So, there are $2^{63}$ possible subsets. Possible events:
  – e.g., Harvard wins $>3$ games
  – e.g., Harvard wins more than UNC, less than Duke
  – e.g., the sum (seeds of ACC teams in final8) is prime

• Effect is that if buy “Harvard to get to semifinal,” then price on Harvard winning round1 goes up.
LMSR market maker

D. Pennock

Event $= E = \text{e.g. Harvard wins } > 3$
Outcome $= o = \text{complete unfolding of tourn}$

Price of $E = \frac{\sum_{o \in E} e^{x_o}}{\sum_{o} e^{x_o}}$

Challenge 1: cannot maintain state explicitly (would need $2^{63}$ numbers)
Challenge 2: need to sum over $2^{63}$ numbers

Yahoo 2010: sample-based approach; but poor user experiences. Prices volatile!

New approach (2013+)

(following Chen et al.)

Proceedings Article

A Combinatorial Prediction Market for the U.S. Elections

MIROSLAV DUDÍK, Microsoft Research, New York City
SEBASTIEN Lahaie, Microsoft Research, New York City
DAVID M. PENNOCK, Microsoft Research, New York City
DAVID ROThSCHILD, Microsoft Research, New York City

We report on a large-scale case study of a combinatorial prediction market. We implemented a back-end pricing engine based on Dudík et al.’s [2012] combinatorial market maker, together with a wizard-like front end to guide users in constructing any of millions of predictions about the presidential, senatorial, and gubernatorial elections in the United States in 2012. Users could create complex combinations of predictions and, as a result, we obtained detailed information about the joint distribution and conditional estimates of
New approach (2013+)

• Don’t work with explicit outcome space

• Introduce “atomic variables”
  – Outcome (FL,Dem), Outcome (FL,Rep)

• Introduce “pairwise variables”
  – Outcome “(FL,Dem) AND (OH,Rep)”

• Triples. Weighted sums.

• Include consistency constraints.

Regulatory Framework
Regulatory Framework

- Pred markets look like gambling (?), which is illegal in many forms in all U.S. states
  - Federal wire act bans use of telephones to accept wagers on sporting events
  - Prevents U.S. banks processing payments to online gambling sites
- Pred markets look a bit like commodity trading (regulated by CFTC)
  - Setting up markets for hedging risks legal, but markets for info aggregation may be gambling

CFTC Complaint

- Nov 26, 2012. The U.S. Commodity Futures Trading Commission (CFTC) today filed a civil complaint in federal district court in Washington, DC, charging Intrade The Prediction Market Limited (Intrade) and Trade Exchange Network Limited (TEN), Irish companies based in Dublin, Ireland, with offering commodity option contracts to U.S. customers for trading, as well as soliciting, accepting, and confirming the execution of orders from U.S. customers, all in violation of the CFTC’s ban on off-exchange options trading.
- From September 2007 to June 25, 2012, Intrade and TEN operated an online “prediction market” trading website, which allowed U.S. customers to trade options products prohibited by the CFTC’s ban on off-exchange options trading. Through the website, Intrade and TEN allegedly unlawfully solicited and permitted U.S. customers to buy and sell options predicting whether specific future events would occur, including whether certain U.S. economic numbers or the prices of gold and currencies would reach a certain level by a certain future date, and whether specific acts of war would occur by a certain future date.
Policy Analysis Market

Real combinatorial markets in Middle East issues

- Open to public, real-money markets
- ~20 nations, 8 quarters, ~5 variables each:
  - Economic, political, military, US actions
- Want many combos (> $2^{500}$ states)

http://en.wikipedia.org/wiki/Policy_Analysis_Market