1. **Sponsored Search Auctions**

   (a) Consider a GSP auction with two positions, three bidders, and position effect 0.3 and 0.2 in positions 1 and 2, respectively (assume quality effect 1.) Provide an example of a valuation profile for which truthful bidding is not a Nash equilibrium. Justify your answer.

   (b) What are the per-click VCG payments in your example in part (a)? Compare them to the GSP prices.

   (c) Give three reasons why GSP rather than VCG is generally used for sponsored search.
2. Combinatorial auctions

(a) A bidder wants to win a New York area wireless spectrum license and a Boston area wireless spectrum license. Briefly, what could go wrong in an auction in which must bid on each item separately, and how is this addressed in a combinatorial auction?

(b) From the perspective of winner determination, why can we assume OR bids?

(c) Express the valuation “I have an additive value for different cars, and want at most three cars” as a succinct OR-of-XOR bid or XOR-of-OR bid. (No need to give the full expression).

(d) Express the valuation “I’m only interested in particular pairs of bikes, but I’d take any number of the pairs I’m interested in” as a succinct OR or XOR bid. (Just show how you’d do this by illustrating this with a few pairs.)
(e) Would you use the OR or XOR logical operator to combine your answers to (c) and (d) and express the valuation for someone interested in cars and bikes, but not a mixture?
3. **Paired-kidney exchange**

Suppose a patient can receive a kidney if the patient’s blood includes all proteins in the blood of the donor (e.g., an A-type can receive O and A, but not from B or AB).

(a) There are four patient-donor pairs, with (patient-donor) blood types \{B-A, AB-B, AB-A, A-A\}. Draw an **undirected** graph to indicate the feasible swaps.

(b) What is the maximum cardinality matching on this input (for swaps)?

(c) How would the graph representation be modified in order to allow for matches that use 3-cycles, in addition to swaps?

(d) How can **altruistic donors** be represented in this graph, and what impact do they have beyond just providing an additional transplant opportunity?

(e) What is a new computational challenge when matching with altruistic donors?
4. **Top-trading cycle mechanism**

(a) Consider seven agents, with preference orders:

1: \( 2 \succeq_1 3 \succeq_1 1 \)
2: \( 3 \succeq_2 4 \succeq_2 5 \succeq_2 2 \)
3: \( 1 \succeq_3 3 \)
4: \( 2 \succeq_4 1 \succeq_4 5 \succeq_4 4 \)
5: \( 4 \succeq_5 5 \)
6: \( 5 \succeq_6 7 \succeq_6 6 \)
7: \( 5 \succeq_7 6 \succeq_7 7 \)

The preference orders are truncated at the point where an agent prefers the house that she already owns. What trades occur in each round of the mechanism?

(b) Explain why the assignment produced by the TTC mechanism is Pareto Optimal. You can argue with respect to this particular example if you choose.

(c) Suppose the *random serial dictator* mechanism (RSD) was used instead, where the houses of the seven agents were first placed into a common ownership pool. What property of TTC is not achieved by RSD?

(d) State two differences between the 3-cycle matching problem in paired-kidney exchange and the matching problem solved in a single round of the TTC mechanism.
5. **Two-sided Matching**

Consider the following preference orders for four firms and four workers (most preferred to least preferred):

- **f1**: w1 w2 w3 w4
- **w1**: f4 f3 f2 f1
- **f2**: w2 w1 w4 w3
- **w2**: f3 f4 f1 f2
- **f3**: w3 w4 w1 w2
- **w3**: f2 f1 f4 f3
- **f4**: w4 w3 w2 w1
- **w4**: f1 f2 f3 f4

(a) Consider the following two stable matchings

- **matching 1**: (f1,w2) (f2,w1) (f3,w3) (f4,w4)
- **matching 2**: (f1,w1) (f2,w2) (f3,w4) (f4,w3)

What is the “join” of these two matchings (where each firm gets its most-preferred worker across both matchings)?

(b) What do you notice about this matching?

Consider the following preference orders for three firms and three workers (most preferred to least preferred):

- **f1**: w1 w2 w3
- **w1**: f1 f2 f3
- **f2**: w1 w3 w2
- **w2**: f1 f2 f3
- **f3**: w1 w2 w3
- **w3**: f1 f3 f2

(c) There’s something about the firms’ preferences that seems to make this matching instance difficult. What is it?

(d) How does stability address this concern in general, and in this example in particular?
6. Peer prediction

(a) Consider a setting with two possible signals \( \ell \) and \( h \). Suppose the output-agreement mechanism is used, paying $1 for agreement and $0 otherwise.

What needs to be true of \( \Pr(X_2 = h \mid X_1 = h) \) and \( \Pr(X_2 = \ell \mid X_1 = \ell) \) for the mechanism to be strictly proper? **Provide a brief justification.**

(b) Suppose the distribution on signals by agents 1 and 2 is

\[
\begin{array}{c|cc}
\text{Player 2} & h & \ell \\
\hline
\text{player 1} & h & 0.5 & 0.2 \\
& \ell & 0.2 & 0.1 \\
\end{array}
\]

What is the design of the 1/prior mechanism in this case? Confirm that the mechanism is strictly proper.

(c) Suppose there are three possible signals \( \{1, 2, 3\} \). Your friend has designed a peer prediction method that satisfies:

\[
\mathbb{E}_{s_2 \sim P(X_2 \mid X_1 = s_1)}[t_1(s_1, s_2)] > \mathbb{E}_{s_2 \sim P(X_2 \mid X_1 = s_1)}[t_1(r_1, s_2)], \quad \forall s_1 \in \{1, 2\}, \forall r_1 \neq s_1.
\]

It does not satisfy this property for \( s_1 = 3 \). Likewise, the same inequality holds for agent 2 and signals \( s_2 \in \{1, 2\} \) (but not \( s_2 = 3 \)).

Will it be a strict best response for player 1 to report her signal truthfully when \( s_1 \in \{1, 2\} \)? Justify your answer.
7. Miscellaneous

(a) Briefly, what two properties of the Blockchain design prevent anyone from easily printing Bitcoins and giving them to themselves?

(b) Denoting a possible outcome \(o_k\) (one of \(k \in \{1, \ldots, m\}\)), write down the definition of strict properness for a scoring rule \(s(q,o)\) that takes belief \(q\) and realized outcome \(o\).

(c) Briefly, what goes wrong with a linear scoring rule and how is this addressed with the logarithmic or quadratic scoring rules?

(d) Give an example of adverse selection, and use that example to explain how a well-functioning reputation system would address the problem of adverse selection.
(e) State two aspects of the “reputation game” model and analysis that make it overly stylized relative to real-world reputation systems.